YBUS Admittance Matrix Formulation

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This document is a description of how to formulate the YBUS admittance matrix. In general, the diagonal terms $Y_{ii}$ are the self admittance terms and are equal to the sum of the admittances of all devices incident to bus $i$. The off-diagonal terms $Y_{ij}$ are equal to the negative of the sum of the admittances joining the two buses. Shunt terms only affect the diagonal entries of the $Y$ matrix. For large systems, $Y$ is a sparse matrix and it is structurally symmetric.

Transmission Lines

Transmission lines run from bus $i$ to bus $j$ and are indexed by $k$. More than one transmission line can go between two buses.

\[
Y_{ij} = \sum_{k} \frac{-1}{r_{ijk} + jx_{ijk}} \quad (1)
\]

\[
Y_{ji} = Y_{ij} \quad (2)
\]

\[
Y_{ii} = -\sum_{j \neq i} Y_{ij} \quad (3)
\]

\[
Y_{jj} = -\sum_{i \neq j} Y_{ji} \quad (4)
\]

where:

$Y_{ij}$: the $ij_{th}$ element in the Y matrix.

$i$: the “from” bus.

$j$: the “to” bus.

$k$: the $k_{th}$ transmission line from $i$ to $j$.

$r_{ijk}$: the resistance of the $k_{th}$ transmission line from $i$ to $j$. 
$x_{ijk}$: the reactance of the $k_{th}$ transmission line from $i$ to $j$.

**Transformers**

For a tap changing and phase shifting transformer, the off-nominal tap value can in general be considered as a complex number $a$, where the tap ratio is $t$ and the phase shift is $\theta$. Transformers are defined similarly to transmission lines and exist on a branch between bus $i$ and bus $j$. Therefore, all transformer elements are subscripted by $i$ and $j$. For simplicity, we assume that only one transformer line exists between two buses and if a transformer is present between two buses then no transmission lines exist between the two buses. The off-nominal tap ratio between buses $i$ and $j$ is defined as $a_{ij} = t_{ij} \ast (\cos \theta_{ij} + j \ast \sin \theta_{ij})$. We can define

$$y_{ij}' = \frac{-1}{r_{ij} + jx_{ij}} \text{ (5)}$$

$$Y_{ij} = \frac{y_{ij}'}{a_{ij}^{*}} \text{ (6)}$$

$$Y_{ji} = \frac{y_{ij}'}{a_{ij}} \text{ (7)}$$

$$Y_{ii} = \frac{y_{ij}'}{|a_{ij}|^2} \text{ (8)}$$

$$Y_{jj} = y_{ij}' \text{ (9)}$$

where:

$Y_{ij}$: the $ij_{th}$ element in the $Y$ matrix.

$i$: the “from” bus.

$j$: the “to” bus.

$r_{ij}$: the resistance of the transformer between $i$ and $j$.

$x_{ij}$: the reactance of the transformer between $i$ and $j$.

$t_{ij}$: the tap ratio between bus $i$ and bus $j$.

$\theta_i$: the phase on bus $i$.

$\theta_j$: the phase on bus $j$.

$\theta_{ij} = \theta_i - \theta_j$: the phase shift from bus $i$ to bus $j$.

$a_{ij}^{*}$: the conjugate of $a_{ij}$.

Given the bus admittance matrix $Y$ for the entire system, the transformer
model can be introduced by modifying the elements of the Y-matrix derived from the transmission lines as follows:

\[ Y_{ij}^{\text{new}} = -\frac{y_{ij}^t}{a_{ij}} \quad (10) \]
\[ Y_{ji}^{\text{new}} = -\frac{y_{ij}^t}{a_{ij}} \quad (11) \]
\[ Y_{ii}^{\text{new}} = Y_{ii} + y_{ij}^t \left| a_{ij} \right|^2 \quad (12) \]
\[ Y_{jj}^{\text{new}} = Y_{jj} + y_{ij}^t \quad (13) \]

Note that since we are assuming that a branch with a transformer on it does not carry any additional transmission lines, the \( Y_{ij}^{\text{new}} \) do not have any contributions from \( Y_{ij} \).

For shunts

Shunts only contribute to diagonal elements. The sources of shunts include:

- Shunt devices located at buses \((g_i^s + jb_i^s)\);
- Transmission line/transformer charging \(b_{ijk}\) (distributed half to each end) from end: \(b_{ik} = 0.5b_{ijk}\); to end: \(b_{jk} = 0.5b_{ijk}\);
- Transmission line/transformer shunt admittance, which is normally a small value: \((g_{ijk}^a + jb_{ijk}^a)\); the shunt admittance contributes the same amount to both ends of the line

Therefore, the general equation for diagonal elements is:

\[ Y_{ii}^{\text{tot}} = -(\sum_{j \neq i} Y_{ij}^{\text{new}}) + g_i^s + jb_i^s + \sum_k (jb_{ki} + g_{ki}^a + jb_{ki}^a) \quad (14) \]
\[ Y_{jj}^{\text{tot}} = -(\sum_{i \neq j} Y_{ji}^{\text{new}}) + g_j^s + jb_j^s + \sum_k (jb_{kj} + g_{kj}^a + jb_{kj}^a) \quad (15) \]
where:

$Y_{ij}^{tot}$: the $ij_{th}$ element in the Y matrix.

$i$: the “from” bus.

$j$: the “to” bus.

$k$: the $k_{th}$ transmission line/transformer from $i$ to $j$.

$g_i + j * b_i$: the shunt at bus $i$.

$b_{ijk}$: the line charging of the $k_{th}$ line.

$b_{ik} = 0.5 * b_{ijk}$: the line charging of the $k_{th}$ line assigned to ”from” end $i$.

$b_{jk} = 0.5 * b_{ijk}$: the line charging of the $k_{th}$ line assigned to ”to” end $j$.

$g^a_{ki} + b^a_{ki} = g^a_{ijk} + j b^a_{ijk}$: the $k_{th}$ line shunt admittance at ”from” end $i$.

$g^a_{kj} + b^a_{kj} = g^a_{ijk} + j b^a_{ijk}$: the $k_{th}$ line shunt admittance at ”to” end $j$.

Thanks!